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Human Skeletal Muscle - Mechanical and Mathematical Models

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Abstract—**This papers addresses new approach to human muscle modeling concerning to the parameters search which will be informative about muscle's condition (relaxed or strengthened, fatigue). The normalizing approach (the dimensionless forms) gave an opportunity to obtain an universal curve, which presents the possibility to compare different muscles' parameters in the relative units. Similar state in different muscles could have similar relative values.**

I. INTRODUCTION

MUSCLE simulation is an important component of human modeling, especially nowadays when we use the powerful computers in support. The knowledge about informative mechanical parameters in muscles are used in medicine to classify different muscles [1], in arts to consider the influence of the musculature on surface form [2- 5]. In sport human muscles' models help to improve the training process' efficiency [3, 4, 6], focusing on different aspects of the muscle conditions: kinematics, dynamics, muscle control strategies and so on.

Our investigation concerns to the sportsmen's muscle visco-elastical properties as a basic parameter for back-loop circuit of control made during the training process. The aim of new model was to find a physical parameters' ratio dependence which allow to compare different muscles in relative numbers. In II part the existing mechanical models are considerate. The part III is for mathematical prove of the proposed new model.

II. MECHANICAL MUSCLE MODELS

A. Mechanical model for muscle activity

There are a lot of models representing contractive and elastic elements of the skeletal muscle, mostly based on classical Hill-type model (Fig. 1 and Fig. 3).

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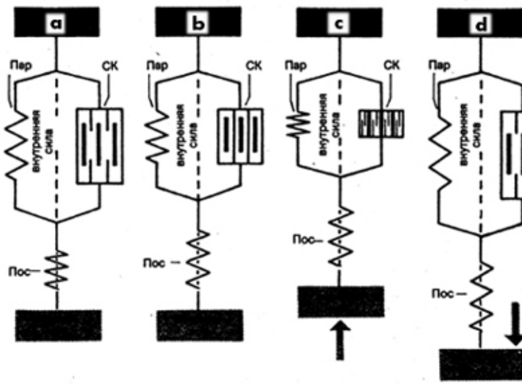

Fig. 1. Mechanical model for muscle activity
a – relax stage, b - static stage, c – dynamic stage, d – stretching stage, where CK – retractor (Contractive Komponent), consists of the muscle fibres or miofiblles
Пap – parallel flexible component, consists of the tube connective tissue layers of muscle fibers (actin - endomysium) and muscular fascicles (myosin). Пoc – sequentially connected flexible component (tendon). Internal force: contraction energy (CK) + preliminary stretching energy (Пap + Пoc)
External force: external resistance.

This model is successfully used in sport medicine [7], the investigators made the conclusion e.g. that when javelin throwing sportsmen practices preliminary stretching energy of the muscles (e.g. uses the wide raise of the javelin), he achieves highest results in compare with the sportsmen which do not use the wide raise of the javelin.

B. Force enhancement and mechanisms of contraction in skeletal muscle

This phenomenon could be explained by the improvement of the preliminary stretching energy made by flexible component and this energy is more in value than the decrease of active retractor strength.

The muscle force for women – is about 6O-100 N/ cm^2 and for men - 70-120 N/ cm^2. The value depends of different factors; concerned to training, as well as to internal fiber construction.

When multiple nerve impulses go frequently one by one, the separate contractions overlap, so there are considered more forcible contraction in 3-4 multiple value of force. This occurrence named as tetanus contraction (Fig. 2).

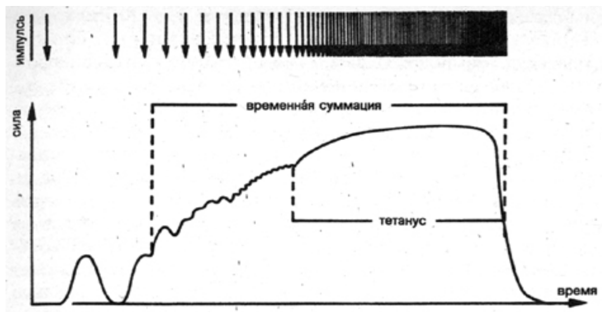

Fig. 2. Scheme of the separate contractions overlap and the occurrence of tetanus contraction.

The range of nerve impulse frequency in 7-10 Hz gives slight strength and force of contraction, 25-30 Hz gives moderate strength and force, and high impulses from 45 Hz give maximum strength and force.

The results of study on force enhancement and mechanisms of contraction in skeletal muscle there are in papers [12]. Force enhancement was evaluated in four conceptually different experimental preparations: in vivo human muscles activated through voluntary contractions; isolated, in situ muscle preparations (typically the cat soleus), isolated single fibers (typically from the lumbrical muscles of the frog), and isolated single myofibrils (from the rabbit psoas). Results show that force enhancement is caused by an active and passive component. The passive component appears to be associated with sarcomere length and have been speculated to originate within the structural protein, titin. The active component of force enhancement appears to be related to the molecular interaction of actin and myosin, and therefore, seems to be intricately linked to the basic mechanism of contraction [12].

C. Muscle fatigue using Hill's model

A model incorporating muscle fatigue has been developed to predict the effect of muscle fatigue on the force-time relationship of skeletal muscle by using the computer program [6].

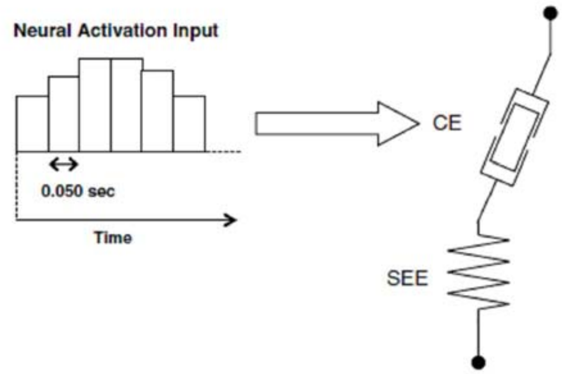

Fig. 3. The Hill-type muscle tendon complex model used in this study. The neural activation input signal was represented by a series of step functions whose duration is 0.050 s.

Differential equations in the incremental form have been implemented into Hill's muscle model. In order to describe the effect of muscle fatigue and recovery on skeletal muscle behaviors, a set of equations in terms of three phenomenological parameters which are a fatigue curve under sustained maximal activation, a recovery curve and an endurance function were developed. With reference to existing models and experimental results, the input parameters for fatigue curve under sustained maximal activation and endurance function were determined. The model has been investigated under an isometric condition. The effects of different shapes of the recovery curves have also been considered in this model. Validation of the model has been performed by comparing the predicted results with the experimental data from an existing literature [6].

D. Proposed mechanical model: Muscle as system "flexible fiber – elastic and viscous bottom layer"

Let us introduce the muscle as a 2D dispersed system "flexible filament – elastic and viscous bottom layer" (Fig.4).

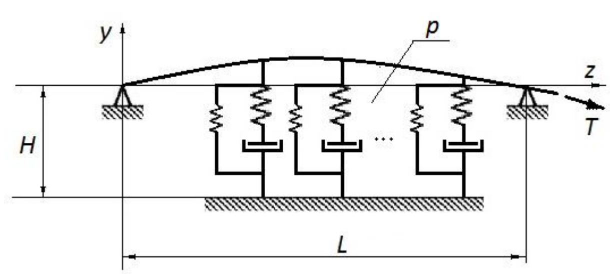

Fig. 4. Mechanical muscle model.

The filament is fixed in both sides and has the tensioning. The flexible filament is an analogue of muscle fibers. Assume the active contraction force which stretch out the filament - T. Assume that the active contraction force in relaxed muscle is vanish, and in tetanus it is maximum. Thus, the muscle ability to contract could be described by active contraction force T. The elastic and viscous bottom layer is an analogue of passive muscle components as well as axillary materials. In this model we neglect the mass of the muscle fibers comparing to inertial passive elements mass. The muscle vibration is a field of investigation which is spread in direction perpendicularly to free surface. The vibration equations for this system in values of idealized mechanical muscle model allow to connect mathematically the main model parameter (muscle tension) with such parameters of vibration process in system as relative period of relaxation oscillations, free oscillation's decrement and frequency. The muscle vibrations could be created by collision momentum perpendicularly to the free surface. The point of collision momentum is in the center of the system, the point of the vibration's registration is known. The decrement of the vibrations is defined from logarithmical relation between two sequential amplitudes of vibration, divided by proper time interval according to the equal:

$$\beta = \ln\frac{A_k}{A_{k+1}}, \quad k = 1, 2, ... \qquad (1)$$

where β – logarithmical vibration decrement; A_k, A_{k+1} – amplitudes of two sequential vibrations, separated from each other by free oscillation period T_1, with angular frequency

$p_1 = 2\pi/T_1$.

III. MATHEMATICAL MODEL

A. *Main forces working in muscle*

In further approximation the muscle will be considered as a plane object with complex structure. Two-dimensional system "flexible filament – elastic and viscous bottom layer" could be presented like a set of parallel filaments connected with elastic-viscid layer (Fig.5).

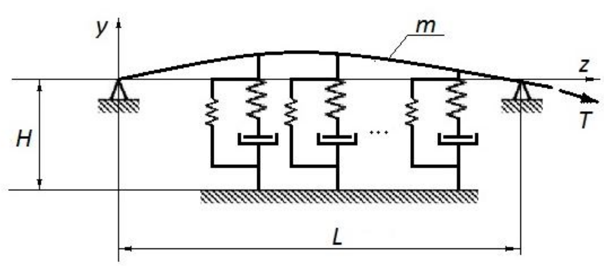

Fig. 5. Mathematical muscle model.

Suppose, that some efficient bottom layer with value H in depth is involved into vibration.

Assume the liner statistical law in axis y for the movement field component, normal to the muscle surface:

$$U(t, y, z) = V(t, z)(1 + y/H) \qquad (2)$$

where

– $U(t, y, z)$ – field component normal to muscle surface for the movement vector,

– $V(t, z)$ – the point of fiber movement, distant to left side z along with axis Oy,

– H – the value in depth of the efficient layer,

– y – axis, directed outside the free surface of muscle,

– z – fiber.

Considering the muscle vibration we suppose, that everything could be aggregate to string's (filament's) dynamic movement investigation, modeling the fibers movements perpendicularly to muscle surface. The part of layer mass taken part in movement, accounted like reduced running mass with intensity m. The value of the connected mass could be estimated in the following way. Use the law for movement field (2). Equate the vis viva energy of the layer in single duration along the strength (filament) E_L to the vis viva energy single length of the filament with the running mass m - E_S

$$E_L(t, z) = \frac{a\rho}{2}\int_{-H}^{0}\left[\frac{\partial U(t, y, z)}{\partial t}\right]^2 dy = \frac{a\rho}{2}\left[\frac{\partial V(t, z)}{\partial t}\right]^2 \int_{-H}^{0}(1 + y/H)^2 dy \qquad (3)$$

$$E_S(t, z) = \frac{m}{2}\left[\frac{\partial V(t, z)}{\partial t}\right]^2$$

where

a - the wide of the bottom layer,

ρ - density of the bottom layer,

$\frac{\partial V(t, z)}{\partial t}$ - velocity along with strength in point z.

After calculations (3), obtain the following

$$E_S = E_L \rightarrow m = \frac{1}{3}\rho a H$$

B. *Operational forces, working with small element of the filament*

The operational forces, working with small element of the filament, shown at the fig. 6.

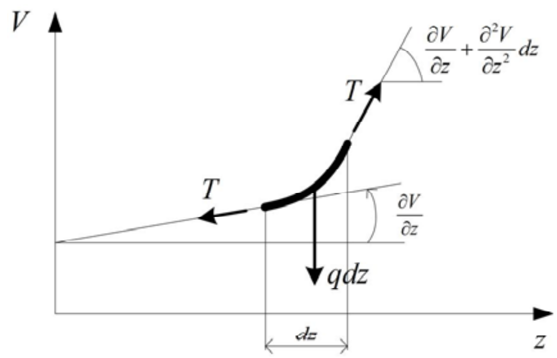

Fig. 6. The operational forces, working with small element of the filament.

The equation for filament small transverse vibrations with elastic-viscid bottom layer has the following view:

$$m\frac{\partial^2 V(t,z)}{\partial t^2} = T\frac{\partial^2 V(t,z)}{\partial z^2} - q \quad (4)$$

where

T - active contraction force, assumed to be constant along the filament, $[T] = \mathrm{N}$;

q - dispersed transverse force working from the side of bottom layer, and directed against the axis y, $[q] = \mathrm{Nm}^{-1}$.

The equation (4) is supplemented with filament fixing and initial conditions. Let us introduce the term of filament's efficient length L. The filament is fixed in both sides:

$$V(t,z)\big|_{z=0} = 0, \quad V(t,z)\big|_{z=L} = 0$$

The dispersed transverse force q is defined through the muscle-layer tension σ, multiplied by efficient wide b

$$q = \sigma b \quad (5)$$

The rheological bottom layer properties could be presented by 3-components model of the viscoelastic object:

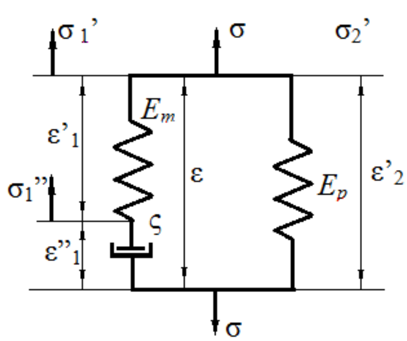

Fig. 7. The 3-components model of the viscoelastic object.

σ – normal tension;

ε – linear deformation;

E_m – elasticity module for serial elastic element;

E_p – elasticity module for parallel elastic element;

η - viscid coefficient for viscid element.

Put the following rate setting for the viscid coefficient

$$\eta = BE_m$$

The value B play role of time constant for the muscle free vibrations process. Thus, the equation for the elastic-viscid object' conditions law according to fig. 7, is following

$$B\frac{d\sigma}{dt} + \sigma = \left(E_p + E_m\right)B\frac{d\varepsilon}{dt} + E_p\varepsilon \quad (6)$$

According to the movement law (2), the equation (6) transferred to

$$B\frac{d\sigma}{dt} + \sigma = \left(E_p + E_m\right)B\frac{\partial V(t,z)}{H\partial t} + E_p\frac{V(t,z)}{H} \quad (7)$$

Consider that the viscous component is considerably small, transfer the equation solution (7) as ternary regular parameter B decomposition

$$\sigma = E_p\frac{V}{H} + E_m B\frac{\partial V(t,z)}{H\partial t} + E_m B^2\frac{\partial^2 V(t,z)}{H\partial t^2} \quad (8)$$

After substitution (5) and (8) in the vibration equation (4) we shell receive the following filament vibration equation

$$M\frac{\partial^2 V(t,z)}{\partial t^2} + d\frac{\partial V(t,z)}{\partial t} + kV(t,z) = T\frac{\partial^2 V(t,z)}{\partial z^2} \quad (9)$$

where $M = m + \frac{bB^2 E_m}{H}$, $k = \frac{bE_p}{H}$, $d = \frac{bBE_m}{H}$

Thus, we have the following list of eight physical parameters, described the muscle dynamic

$$\{m, b, B, H, T, E_p, E_m, L\}$$

C. Scale choosing

Let us use three scales: the velocity scale V_*, $[V_*] = \text{m.s}^{-1}$, the length scale Z_*, $[Z_*] = \text{m}$ and the time scale T_*, $[T_*] = \text{s}$ –

$$V = V_*\xi, \quad z = Z_*\zeta, \quad t = T_*\tau \tag{10}$$

Put (10) in (9) and divide by MV_*/T_*^2 :

$$\frac{\partial^2\xi(\tau,\zeta)}{\partial\tau^2} + \frac{dT_*}{M}\frac{\partial\xi(\tau,\zeta)}{\partial\tau} + \frac{kT_*^2}{M}\xi(\tau,\zeta) = \\ = \frac{TT_*^2}{MZ_*^2}\frac{\partial^2\xi(\tau,\zeta)}{\partial\zeta^2}$$

Put the length scale equal to the filament efficient length $Z_* = L$

To choose the time scale, suppose that in case of dissipation absence in bottom layer the first form of natural vibrations

$$\xi(\tau,\zeta) = q(\tau)\sin(\pi\zeta) \tag{11}$$

have the oscillation frequency equal to one

$$\left(k + \pi^2 T/L^2\right)\frac{T_*^2}{M} = 1 \tag{12}$$

Thus, obtain from (12) the value for time scale:

$$T_* = \sqrt{\frac{M}{k + \left(\pi^2 T / L^2\right)}}$$

Suppose, that in case of existing dissipation the movement law come approximately (11), then for the amplitude function $q(\tau)$ the vibration equation in dimensionless form could be written like

$$\frac{d^2q(\tau)}{d\tau^2} + 2n\frac{dq(\tau)}{d\tau} + q(\tau) = 0 \tag{13}$$

where the normal damping coefficient defined as

$$n = \frac{d}{2\sqrt{M\left(k + \pi^2 T / L^2\right)}}$$

Note that the normal damping coefficient n depends on muscle fibers tension T .

General equation solution for filament free vibrations (13) has following view

$$q(\tau) = Ae^{-n\tau}\cos\left(\sqrt{1-n^2}\tau + \varphi\right)$$

The period of free vibrations is equal to

$$T_1 = \frac{2\pi}{\sqrt{1-n^2}}T_*$$

The normal damping coefficient n (14) links to logarithmic decrement β (1) by ratio

$$n = \frac{\beta}{2\pi\sqrt{1+\beta^2}}$$

When $\beta \ll 1$ instead (15) the dependence is used

$$n = \frac{\beta}{2\pi}$$

IV. Reduction to the Dimensionless Form

The value of the logarithmic decrement is a measurable value and it could be used to define the two dimensionless complexes Π_d, Π_T

$$\Pi_d = \frac{d}{2\sqrt{Mk}}, \quad \Pi_T = \frac{\pi^2 T}{kL^2} \tag{17}$$

From(14), (16), (17) follows correlation between the normal damping coefficient and dimensionless complexes Π_d, Π_T

$$\beta = \frac{2\pi\Pi_d}{\sqrt{1+\Pi_T}} \tag{18}$$

If put that in relaxed conditions the muscle filament strength (tension) is to be zero – $\Pi_T = 0$, – thus according to the logarithmic decrement definition as β_0 the dimensionless complexes Π_d is defined as

$$\Pi_d = \frac{\beta_0}{2\pi} \tag{19}$$

During the logarithmic decrement measuring in strengthen muscle β the dimensionless complexes Π_T is defined as follows(18), (19)

$$(14)\Pi_T = \frac{\beta_0^2}{\beta^2} - 1 \tag{20}$$

The parameter Π_T is informative and it could be used for estimation of the level of muscle excitation, depended of human condition. The figure 8 shows the dependence

$$\Pi_T(x), \quad x = \frac{\beta}{\beta_0}$$

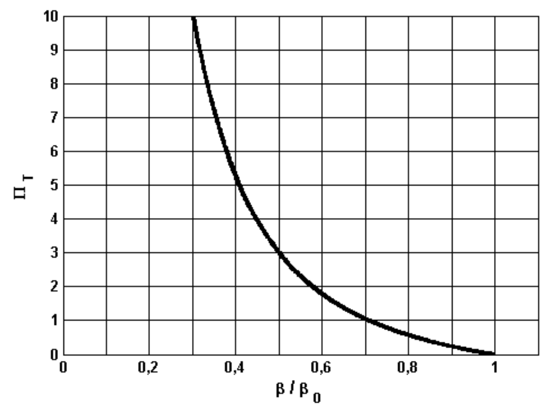

Fig. 8. The dimensionless complexes Π_T dependence.

This curve is universal in the sense that it gives a possibility to compare different muscles properties in relative numbers. The identical values of the Π_T corresponds to identical (similar) muscles conditions, possibilities, though the physical parameters, such as $\{T,k,L\}$ (17) could be different.

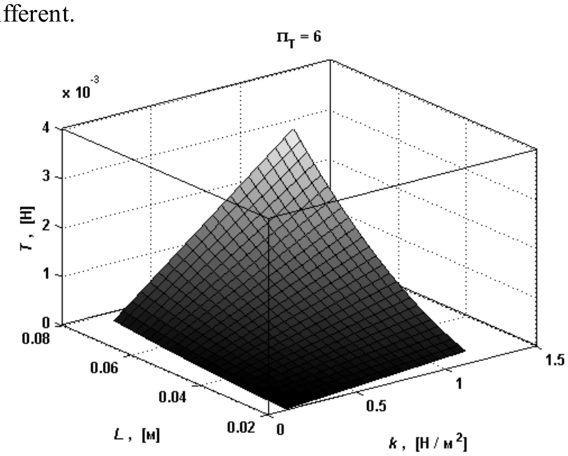

Fig. 9 - The surface of active contraction force value $T(k,L)$ in case if $\Pi_T = 6$.

At the figure 9 the surface $T(k,L)$ is shown correspondent to the tension dimensionless parameter value $\Pi_T = 6$, according to (17). That is, for example, the muscle with more long filaments needs more rigid bottom layer to achieve same level of tension, and so on.

The measure of the muscle surface logarithmic decrement could be realized by several physical ways. Particularly [13, 14], the myotonometry method uses the equipment, consist of shooting mass and acceleration gage, placed in fixed point from the point of impulse application. The measuring result is a dependence between motion and time: $V_{\text{experimental}}(t,z_0)$, where z_0- coordinate of acceleration gage's fixation.

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